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Gravitational Perturbation of Garfinkle-Horowitz-Strominger Dilaton Black Hole and Quasinormal Modes

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Abstract We research gravitational perturbation of Garfinkle-Horowitz-Strominger dilaton black hole and its quasinormal modes by using WKB approach proposed by Schutz, Will, Iyer and Konoplya. The quasinormal frequency with different angular momentum l is calculated in this paper. Our results show that, as the charge parameter b increase, both the real part and the absolute value of imaginary part of quasinormal frequency also increase, which means that the effect of charge in Garfinkle-Horowitz-Strominger dilaton background spacetime lead to higher frequency gravitational wave and the quasinormal modes damp at a rapider rate.

Keywords Garfinkle-Horowitz-Strominger dilaton black hole · Quasinormal modes · Gravitational wave · Six-order WKB approach

1 Introduction

Gravitational wave is one of the most interesting, important and vital research subject in modern theory physics. In quantum field theory, gravitational wave is the classical scenic of the quantum field with spin 2, but it is a result of weak field approximation in general relativity. Therefore, the research about gravitational wave could be the connection point between quantum field and general relativity and the breakthrough point of modern physics' development. At present, studying the quantum gravity is viewed as a realistic method to solve several problem in theory physics, so it is inevitable to investigate quantum effect of black hole firstly because the physical behavior near the horizon is very simple.

In the study of the quantum effect of black hole, a significant property of black hole is quasinormal modes, which follow the perturbation of black hole. Different perturbation (scalar, electromagnetic, Dirac, gravitational perturbation and so on) could make different quasinormal frequency, which generally is a plural and the real and imaginary part show the real frequency and the damping rate of quasinormal modes. Being similar to the black hole

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solution, gravitational perturbation also is the solution of Einstein field equation in nature, so the research about gravitational quasinormal modes is of concern very much, it help to learn the inherent characteristic of gravity object and understand the deeper physics mechanism. Unfortunately, most of quasinormal modes' equation is hard to obtain the analytic solutions, so people have proposed several numerical method to calculate the quasinormal frequency [1–37], and WKB approach is available, simple and reliable method. This numerical way is proposed by Shchutz and Will firstly [38], and has been now developed as six-order correction by Iyer, Will [39] and Konoplya [40–44].

We in this paper will study the gravitational perturbation and quasinormal modes of Garfinkle-Horowitz-Strominger dilaton black hole, which is charged black hole in string theory. The equation of gravitational perturbation of this black hole is derived in Sect. 2, and we compute the quasinormal frequency by means of six-order WKB approach in Sect. 3. Finally, the Sect. 4 include some discussion and conclusion.

2 Gravitational Perturbation of Garfinkle-Horowitz-Strominger Dilaton Black Hole

In string theory, the uncharged spacetime, except the region near the singularity, can be described by Schwarzschild solution, when the mass of black hole is larger Planck mass. But the Reissner-Nordström solution fail to describe a charged black hole in string theory, and a charged string black hole is proposed by Garfinkle, Horowitz and Strominger [45, 46]. After proper transformation of coordinates, the metric of charged black hole is given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{k(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where

$$\begin{aligned} f(r) &= 1 - \frac{2M}{b + \sqrt{b^2 + r^2}} \\ k(r) &= 1 - \frac{2M}{b + \sqrt{b^2 + r^2}} \frac{r^2}{r^2 + b^2} \end{aligned} \tag{2}$$

In (2), M is mass parameter, and $b = \frac{Q^2}{2M}$ describe the charged effect.

On the other hand, the gravitational perturbation in curved background can written as

$$\delta R_{\mu\nu} = -\delta\Gamma^{\beta}_{\mu\nu;\beta} + \delta\Gamma^{\beta}_{\mu\beta;\nu} = 0 \tag{3}$$

where

$$\delta\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\nu}(h_{\beta\nu;\gamma} + h_{\gamma\nu;\beta} - h_{\beta\gamma;\nu}) \tag{4}$$

According to the Regge-Wheeler gauge [47], the odd gravitational perturbation can be given by

$$h_{\mu\nu} = \begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{vmatrix} e^{-i\omega t} \left(\sin\theta \frac{\partial}{\partial\theta} \right) P_l(\cos\theta) \tag{5}$$

These formula directly result in

$$\begin{aligned} \delta R_{13}=0 \quad &\Rightarrow \quad 2ir\omega h_0+h_1(r^2\omega^2+rk(r)f(r)'-f(r)(l^2+l-2k(r)-rk'))=ir^2\omega h_0' \\ \delta R_{23}=0 \quad &\Rightarrow \quad 2i\omega h_0+h_1(k(r)f(r)'+f(r)k(r)')+2f(r)k(r)h_1'=0 \end{aligned} \tag{6}$$

By redefining the variable as $\Phi=\frac{\sqrt{f(r)k(r)}}{r}h_1(r)$, the radial gravitational perturbation equation of this black hole with Schrödinger-like form can be obtained

$$\frac{d^2\Phi}{dr_*^2}+(\omega^2-V(r))\Phi=0 \tag{7}$$

where

$$V(r)=\frac{fl(l+1)}{r^2}-\frac{3(fk)'}{2r}, \tag{8}$$

and $r_*=\int\frac{dr}{\sqrt{f(r)k(r)}}$.

The equation imply that gravitational perturbation of black hole can produce gravitational wave. In fact, the gravitational wave from black hole is an important object in currently gravitational wave detecting, and the signals from black hole in theory can carry the information of black holes, so the research of gravitational quasinormal modes is useful. In black hole thermodynamics near the event horizon, the Hawking radiation is from (7) with $V(r)=0$, so the equation is very simple [48–66]. However the simple equation in the study of quasinormal modes is not true, so that it is difficulty to calculate the quasinormal modes frequency analytically. We will use six-order WKB approach to study the quasinormal modes of Garfinkle-Horowitz-Strominger dilaton black hole in Sect. 3.

3 Numerical Results via Six-Order WKB Approach

Base on the works of Shchutz, Will and Iyer, Konoplya calculated the six-order WKB approach of quasinormal modes. According to their works, from (7) the quasinormal modes frequency ω is determined by

$$\omega=\sqrt{-iV+\sqrt{2V''}\left(n+\frac{1}{2}+\Lambda_2+\Lambda_3+\Lambda_4+\Lambda_5+\Lambda_6\right)}\Bigg|_{r=r_0} \tag{9}$$

where the r_0 is the maximum value of $V(r)$, and Λ_n represent the n-th order correction, and the concrete form of the corrections can be found in Ref. [38, 39] and [40].

The results from six-order WKB approach are very reliable when the parameters n and l are not too large, so it is safe to compute quasinormal modes frequency by using this method. First of all, we draw the gravitational quasinormal modes equation's potential $V(r)$. As we have seen, as b increase, the peak value of potential increases but the location of peak r_0 decrease.

In Fig. 1, we plot the potential of electromagnetic QMNs' equation with different α in Hořava-Lifshitz spacetime. It show that the peak value of potential increases but the location of peak r_0 decrease as α increase.

From Tables 1–4, we find that the real parts of QNF increase but the imaginary parts decrease as the b increase in Garfinkle-Horowitz-Strominger dilaton spacetime. It show the effect of charged in string theory black hole make black hole radiate higher frequency gravitational wave with rapider rate. It is a particular "song" of charged black hole in string theory.

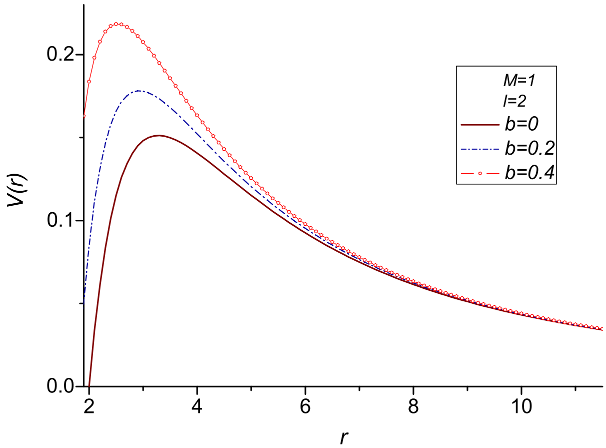

Fig. 1 The potential of gravitational quasinormal modes of Garfinkle-Horowitz-Strominger dilaton black hole with $b = 0$, $b = 0.2$ and $b = 0.4$ for the parameters $M = 1$ and $l = 2$

Table 1 Quasinormal modes frequency of gravitational perturbation of Garfinkle-Horowitz-Strominger dilaton black hole with $l = 2$ for black hole's mass $M = 1$

α	ω $(n = 0)$	ω $(n = 1)$	ω $(n = 2)$	ω $(n = 3)$
0	$0.37362 - 0.08889i$	$0.34630 - 0.27348i$	$0.29852 - 0.47756i$	$0.24172 - 0.70960i$
0.2	$0.40771 - 0.09101i$	$0.38414 - 0.27948i$	$0.34413 - 0.48554i$	$0.29899 - 0.71516i$
0.4	$0.45439 - 0.09266i$	$0.43517 - 0.28327i$	$0.40287 - 0.48774i$	$0.36707 - 0.70929i$
0.6	$0.52482 - 0.09340i$	$0.51010 - 0.28459i$	$0.48461 - 0.49023i$	$0.45747 - 0.72181i$

Table 2 Quasinormal modes frequency of gravitational perturbation of Garfinkle-Horowitz-Strominger dilaton black hole with $l = 3$ for black hole's mass $M = 1$

α	ω $(n = 0)$	ω $(n = 1)$	ω $(n = 2)$	ω $(n = 3)$
0	$0.59944 - 0.09270i$	$0.58264 - 0.28129i$	$0.55159 - 0.47905i$	$0.51110 - 0.69047i$
0.2	$0.64877 - 0.09489i$	$0.63360 - 0.28756i$	$0.60564 - 0.48850i$	$0.56935 - 0.70164i$
0.4	$0.71490 - 0.09660i$	$0.70198 - 0.29226i$	$0.67820 - 0.49489i$	$0.64740 - 0.70768i$
0.6	$0.81150 - 0.09699i$	$0.80157 - 0.29280i$	$0.78324 - 0.49376i$	$0.75939 - 0.70227i$

Table 3 Quasinormal modes frequency of gravitational perturbation of Garfinkle-Horowitz-Strominger dilaton black hole with $l = 4$ for black hole's mass $M = 1$

α	ω $(n = 0)$	ω $(n = 1)$	ω $(n = 2)$	ω $(n = 3)$
0	$0.80918 - 0.09416i$	$0.79663 - 0.28433i$	$0.77270 - 0.47990i$	$0.73967 - 0.68390i$
0.2	$0.87334 - 0.09630i$	$0.86195 - 0.29059i$	$0.84024 - 0.48974i$	$0.81032 - 0.69646i$
0.4	$0.95862 - 0.09795i$	$0.94884 - 0.29527i$	$0.93021 - 0.49672i$	$0.90454 - 0.70455i$
0.6	$1.08186 - 0.09815i$	$1.07431 - 0.29551i$	$1.05990 - 0.49592i$	$1.03996 - 0.70105i$

Table 4 Quasinormal modes frequency of gravitational perturbation of Garfinkle-Horowitz-Strominger dilaton black hole with $l = 5$ for black hole's mass $M = 1$

α	ω $(n = 0)$	ω $(n = 1)$	ω $(n = 2)$	ω $(n = 3)$
0	$1.01230 - 0.09487i$	$1.00222 - 0.28582i$	$0.98269 - 0.48033i$	$0.95496 - 0.68054i$
0.2	$1.09113 - 0.09698i$	$1.08197 - 0.29203i$	$1.06422 - 0.49029i$	$1.03903 - 0.69370i$
0.4	$1.19550 - 0.09858i$	$1.18762 - 0.29666i$	$1.17234 - 0.49748i$	$1.15066 - 0.70264i$
0.6	$1.34557 - 0.09868i$	$1.33946 - 0.29674i$	$1.32763 - 0.49683i$	$1.31079 - 0.70014i$

4 Conclusion

In this paper, we have study the gravitational perturbation and quasinormal modes of Garfinkle-Horowitz-Strominger dilaton black hole. As a charged black hole in string theory, the quasinormal modes behavior of this black hole is different from Reissner-Nordström case. Both the real part and the absolute value of imaginary part of quasinormal frequency increase with the charged effect increase.

In fact, the research about quasinormal modes is important in experiment, astroobservation and theory. In experiment and astroobservation, we can decide "whether the radiation is from black hole" and "which kind of black hole emit this radiation". In theory, people could guess the property of black hole by the quasinormal modes frequency. Especially, a new viewpoint about the calculation of area spectra is put forward recently [67–70]. This method emphasize that the area spectra of black hole is mainly determined by imaginary part of quasinormal modes frequency, because the contribution of the highly damped modes is mainly from imaginary part. This viewpoint provide a new train of thought to investigate the quasinormal modes in theory.

On the other hand, as already noted, gravitational wave is weak field solution in general relativity theory, but describe the classical analog of particles with spin 2 in quantum field theory, and the form of this weak field equation is very similar to classical Klein-Gordon equation, Maxwell equation and Dirac equation. This fact maybe imply that these classical field equations in quantum field are also the weak field approximation of undiscovered theory, so that the quantization of these field is simpler than gravity field. A successful equation of the theory of everything maybe a strongly non-linear differential equation. In the process to find this theory, it is obvious that the detecting and research of gravitational wave play an important role. A possible method for proving this assumption is proposing the second-order scalar, Maxwell and Dirac perturbation equations. For this purpose, we can firstly obtain the second-order approximate gravitational weak field equation by coordinates condition $\Box x_\mu = 0$, Einstein field equation and equations for geodesics, which are complete equations to describe gravitational interaction. And then, considering this fact that first-order weak gravitational field equation can be obtain by the substitute the linear scalar or Maxwell field's variable ϕ or A_μ for second order tensor $h_{\mu\nu}$, we can change the tensor $h_{\mu\nu}$ in second-order weak gravitational field equation as ϕ or A_μ to obtain the second-order correctional "Klein-Gordon equation" or "Maxwell equation", and further obtain the second-order correctional "Dirac equation". These second-order correctional field must be non-linear, and, specially, the non-linear effect of electromagnetic wave can be inspected and verified by nonlinear optics. These researches could help to bridge between the gravitational interaction and other interactions, and it is no other than our further's work.

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